

# Towards isolation of new physics in $B_s^0$ - $\bar{B}_s^0$ mixing

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**Abstract.** In many extensions of the standard model, new physics is possible to contribute significantly to  $B_s^0$ - $\bar{B}_s^0$  mixing. We show that the effect of new physics, both its phase information and its magnitude, can be isolated from measurements of  $CP$  asymmetries in the semileptonic  $B_s$  transitions and in some nonleptonic  $B_s$  decays into hadronic  $CP$  eigenstates. We also find that the rates of  $CP$ -forbidden decays at the  $\Upsilon(5S)$  resonance are not suppressed by the large  $B_s$  mass difference, thus they can be used to extract the  $CP$ -violating phase of new physics in either  $B_s^0$ - $\bar{B}_s^0$  or  $K^0$ - $\bar{K}^0$  mixing.

## 1 Introduction

Recently some experimental efforts have been made to measure the  $B_s^0$ - $\bar{B}_s^0$  mixing parameter  $x_s$  (corresponding to the mass difference between two  $B_s$  mass eigenstates), which is anticipated to be very large in the standard model (SM) [1]. This stimulates some theoretical or phenomenological interest for a more delicate study of the problem of  $B_s^0$ - $\bar{B}_s^0$  mixing and  $CP$  violation (see, e.g., [2–4]). In particular, the other  $B_s^0$ - $\bar{B}_s^0$  mixing parameter  $y_s$  (corresponding to the width difference between two  $B_s$  mass eigenstates) is predicted to be about 0.1 within the SM, a value making the measurement of untagged  $B_s$  data samples meaningful in the future experiments [2, 5].

The  $B_s^0$ - $\bar{B}_s^0$  mixing system is a good place for the exploration of new physics (NP) beyond the SM [6]. For instance, observation of a sufficiently small  $x_s$ , an asymmetry between the semileptonic  $B_s^0$  and  $\bar{B}_s^0$  decays, or a significant  $CP$ -violating signal in  $B_s^0$  vs  $\bar{B}_s^0 \rightarrow \psi\phi$  modes should imply the existence of NP in  $B_s^0$ - $\bar{B}_s^0$  mixing. In most extensions of the SM, NP is expected to contribute only to the mass difference of  $B_s$  mesons [7]. Any NP does not significantly affect the direct decays of  $B_s$  mesons via the tree-level  $W$ -mediated channels, thus the total decay width  $\Gamma$  remains to amount to its SM value as a good approximation. However, the effect of NP can appear in both  $x_s$  and  $y_s$ , and it may also give rise to a significant  $CP$ -violating phase, which is primarily vanishing in the SM. Consequently it is possible to isolate NP in  $B_s^0$ - $\bar{B}_s^0$  mixing through the measurement of  $x_s$ ,  $y_s$  as well as  $CP$  violation in both semileptonic and nonleptonic  $B_s$  transitions.

Up to now, the possibility to extract the *phase* of NP from  $CP$  asymmetries in some  $B_s$  decays has been considered (see, e.g., [5, 8]). In contrast, little attention has been paid to determining the *magnitude* of NP in  $B_s^0$ - $\bar{B}_s^0$  mixing, a crucial quantity for one to get at the specific non-standard electroweak model. In this work we shall discuss a few instructive approaches towards the isolation of

NP in  $B_s^0$ - $\bar{B}_s^0$  mixing, both its phase information and its magnitude.

The remaining parts of this paper are organized as follows: in Sect. 2 we describe a parametrization of NP in different  $B_s^0$ - $\bar{B}_s^0$  mixing quantities. The NP effects on  $CP$  asymmetries in semileptonic and nonleptonic decays of  $B_s$  mesons are discussed in Sects. 3 and 4, respectively. Section 5 is devoted to extracting the phase information of NP from the  $CP$ -forbidden  $B_s^0$ - $\bar{B}_s^0$  transitions. Finally some concluding remarks are given in Sect. 6.

## 2 NP in $B_s^0$ - $\bar{B}_s^0$ mixing

In the assumption of  $CPT$  invariance, the mass eigenstates of  $B_s^0$  and  $\bar{B}_s^0$  mesons can be written as

$$\begin{aligned} |B_1\rangle &= p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \\ |B_2\rangle &= p|B_s^0\rangle - q|\bar{B}_s^0\rangle, \end{aligned} \quad (1)$$

where  $p$  and  $q$  are complex mixing parameters. In terms of off-diagonal elements of the  $2 \times 2$   $B_s^0$ - $\bar{B}_s^0$  mixing Hamiltonian  $\mathbf{M} - i\mathbf{\Gamma}/2$ , we express the ratio  $q/p$  as

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}. \quad (2)$$

The mass and width differences of  $B_1$  and  $B_2$  are defined by  $\Delta m \equiv |m_1 - m_2|$  and  $\Delta\Gamma \equiv |\Gamma_1 - \Gamma_2|$ , respectively. Furthermore, we define  $m \equiv (m_1 + m_2)/2$  and  $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$ . It is also convenient to use the following dimensionless parameters to describe  $B_s^0$ - $\bar{B}_s^0$  mixing:

$$x_s \equiv \frac{\Delta m}{\Gamma}, \quad y_s \equiv \frac{\Delta\Gamma}{2\Gamma}. \quad (3)$$

The present experimental lower bound for  $x_s$  is  $x_s \geq 15$  [1], implying that  $\Delta m \gg \Delta\Gamma$  or  $|M_{12}| \gg |\Gamma_{12}|$  generally

holds. Consequently one gets

$$\Delta m = 2|M_{12}| \quad (4)$$

to a high degree of accuracy. Then the width difference  $\Delta\Gamma$  reads [5]

$$\begin{aligned} \Delta\Gamma &= \frac{4|\operatorname{Re}(M_{12}\Gamma_{12}^*)|}{\Delta m} \\ &= 2|\Gamma_{12}| \cdot |\cos\phi_m|, \end{aligned} \quad (5)$$

where  $\phi_m \equiv \arg(-M_{12}/\Gamma_{12})$ . In addition, calculations based on the SM yields  $y_s \approx 0.08$  [3]. This value will always be reduced if  $B_s^0$ - $\bar{B}_s^0$  mixing receives  $CP$ -violating contributions from NP (i.e.,  $\phi_m \neq 0$ ). Thus  $x_s/y_s > 10^2$  is true both within and beyond the SM.

Now we assume that NP contributes to  $\Delta m$  through  $M_{12}$ , which can be parametrized as

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}} = M_{12}^{\text{SM}}(1 + ze^{i\theta}) \quad (6)$$

with  $z \equiv |M_{12}^{\text{NP}}/M_{12}^{\text{SM}}|$  and  $\theta \equiv \arg(M_{12}^{\text{NP}}/M_{12}^{\text{SM}})$ . The effect of NP on  $\Gamma_{12}$  is anticipated to be negligibly small, hence  $\Gamma_{12} = \Gamma_{12}^{\text{SM}}$  and  $\Gamma = \Gamma^{\text{SM}}$  hold as a good approximation [6]. Since  $M_{12}^{\text{SM}}$  and  $\Gamma_{12}^{\text{SM}}$  are dominantly proportional to the Cabibbo-Kobayashi-Maskawa matrix elements  $V_{tb}V_{ts}^*$  and  $V_{cb}V_{cs}^*$ , respectively,  $\phi_m = 0$  turns out to be an excellent approximation in the Wolfenstein phase convention. The presence of NP in  $M_{12}$  modifies the SM value of  $\phi_m$  in the following manner:

$$\begin{aligned} \phi_m &= \arg(1 + ze^{i\theta}) \\ &= \arctan\left[\frac{z \sin\theta}{1 + z \cos\theta}\right]. \end{aligned} \quad (7)$$

The  $B_s^0$ - $\bar{B}_s^0$  mixing observables  $x_s$  and  $y_s$ , in the existence of NP, read

$$x_s = x_s^{\text{SM}}\sqrt{1 + z^2 + 2z \cos\theta} \quad (8)$$

and

$$y_s = y_s^{\text{SM}}\sqrt{1 - \frac{z^2 \sin^2\theta}{1 + z^2 + 2z \cos\theta}}, \quad (9)$$

where  $x_s^{\text{SM}}$  and  $y_s^{\text{SM}}$  are (in principle) calculable from the SM. Finally, the ratio  $q/p$  can be expressed as

$$\begin{aligned} \frac{q}{p} &= \sqrt{\frac{(1 + ze^{-i\theta}) + iy_s^{\text{SM}}/x_s^{\text{SM}}}{(1 + ze^{+i\theta}) + iy_s^{\text{SM}}/x_s^{\text{SM}}}} \\ &\approx e^{-i2\phi_m} \left[ 1 - \frac{z \sin\theta}{1 + z^2 + 2z \cos\theta} \cdot \frac{y_s^{\text{SM}}}{x_s^{\text{SM}}} \right] \end{aligned} \quad (10)$$

in the next-to-leading order approximation. Note that  $\phi_m$  is indeed a function of  $z$  and  $\theta$ , as given in (7). Clearly  $\arg(q/p) = -2\phi_m$  is a good approximation when we discuss  $CP$  violation in nonleptonic  $B_s$  decays. The small correction term in  $q/p$ , which is proportional to  $z \sin\theta$

and  $y_s^{\text{SM}}/x_s^{\text{SM}}$ , is crucial for an asymmetry between the semileptonic  $B_s^0$  and  $\bar{B}_s^0$  transitions (see Sect. 3).

One can see, from (9), that  $y_s \leq y_s^{\text{SM}}$  is always true. In contrast,  $x_s \leq x_s^{\text{SM}}$  takes place only if  $\theta \in [-\pi, -\pi/2]$  or  $[\pi/2, \pi]$ , corresponding to the destructive interference between  $M_{12}^{\text{NP}}$  and  $M_{12}^{\text{SM}}$ . Provided  $z \gg 1$  (i.e., NP dominates  $B_s^0$ - $\bar{B}_s^0$  mixing), we obtain  $x_s \approx zx_s^{\text{SM}}$  and  $y_s \approx y_s^{\text{SM}} \cos\theta$ . In this case,  $q/p = e^{-i2\phi_m}$  holds to a higher degree of accuracy.

### 3 NP effect in semileptonic $B_s$ decays

Now we examine the NP effect on  $CP$  violation in semileptonic transitions of  $B_s$  mesons. Assuming both the  $\Delta Q = \Delta B$  rule and  $CPT$  invariance, one is able to get the time-integrated wrong-sign events of semileptonic  $B_s$  decays as follows [9,10]:

$$\begin{aligned} N^-(B_s^0) &= n_0 \left[ \frac{1}{1 - y_s^2} - \frac{1}{1 + x_s^2} \right] \left| \frac{q}{p} \right|^2, \\ N^+(\bar{B}_s^0) &= n_0 \left[ \frac{1}{1 - y_s^2} - \frac{1}{1 + x_s^2} \right] \left| \frac{p}{q} \right|^2, \end{aligned} \quad (11)$$

where  $n_0$  is a normalization factor proportional to the rate of the right-sign semileptonic  $B_s$  decay. The  $CP$  asymmetry between the above two processes turns out to be

$$\mathcal{A}_{\text{SL}} \equiv \frac{N^+(\bar{B}_s^0) - N^-(B_s^0)}{N^+(\bar{B}_s^0) + N^-(B_s^0)} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}, \quad (12)$$

apparently irrelevant to the mixing parameters  $x_s$  and  $y_s$ .

Such an asymmetry can also be obtained from the same-sign dilepton events of coherent  $B_s^0\bar{B}_s^0$  decays at the  $\Upsilon(5S)$  resonance, whose rates are given by [10,11]

$$\begin{aligned} N_C^{--}(B_s^0\bar{B}_s^0) &= n_C \left[ \frac{1 + Cy_s^2}{(1 - y_s^2)^2} - \frac{1 - Cx_s^2}{(1 + x_s^2)^2} \right] \left| \frac{q}{p} \right|^2, \\ N_C^{++}(B_s^0\bar{B}_s^0) &= n_C \left[ \frac{1 + Cy_s^2}{(1 - y_s^2)^2} - \frac{1 - Cx_s^2}{(1 + x_s^2)^2} \right] \left| \frac{p}{q} \right|^2, \end{aligned} \quad (13)$$

in which  $C$  ( $= \pm 1$ ) is the charge-conjugation parity of the  $B_s^0\bar{B}_s^0$  pair, and  $n_C$  denotes the normalization factor proportional to the rates of right-sign semileptonic decays of  $B_s^0$  and  $\bar{B}_s^0$  mesons. Obviously, we arrive at the same  $CP$  asymmetry  $\mathcal{A}_{\text{SL}}$ :

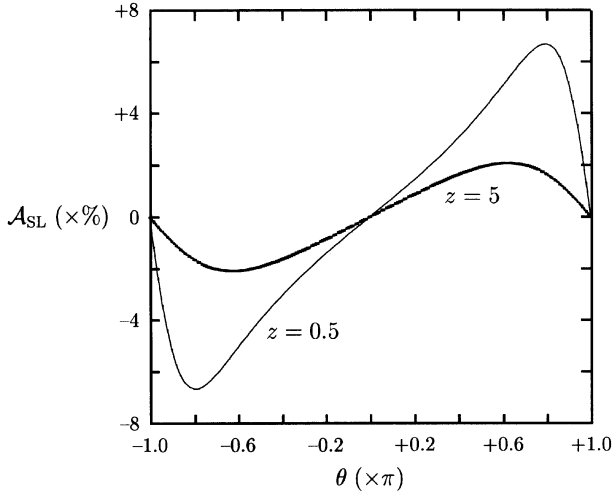
$$\frac{N_C^{++}(B_s^0\bar{B}_s^0) - N_C^{--}(B_s^0\bar{B}_s^0)}{N_C^{++}(B_s^0\bar{B}_s^0) + N_C^{--}(B_s^0\bar{B}_s^0)} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}, \quad (14)$$

for both  $C = -1$  and  $C = +1$  cases.

Taking  $|M_{12}| \gg |\Gamma_{12}|$  into account, one can obtain  $\mathcal{A}_{\text{SL}} \approx \operatorname{Im}(\Gamma_{12}/M_{12})$  as a good approximation. By use of (10), we find that  $\mathcal{A}_{\text{SL}}$  explicitly reads

$$\mathcal{A}_{\text{SL}} \approx \frac{2z \sin\theta}{1 + z^2 + 2z \cos\theta} \cdot \frac{y_s^{\text{SM}}}{x_s^{\text{SM}}}. \quad (15)$$

Within the SM (i.e.,  $z = 0$  and  $\theta = 0$ ),  $\mathcal{A}_{\text{SL}}$  is vanishingly small. In view of the fact  $y_s^{\text{SM}}/x_s^{\text{SM}} \sim O(10^{-2})$ , we believe



**Fig. 1.** Illustrative plot for changes of the semileptonic  $CP$  asymmetry  $\mathcal{A}_{\text{SL}}$  with  $\theta$ , where  $y_s^{\text{SM}}/x_s^{\text{SM}} = 0.05$  and  $z = 0.5$  or  $z = 5$  have been taken

that the presence of NP in  $B_s^0$ - $\bar{B}_s^0$  mixing could enhance the asymmetry  $\mathcal{A}_{\text{SL}}$  to the level of  $10^{-3} - 10^{-2}$ . For the purpose of illustration, we plot changes of  $\mathcal{A}_{\text{SL}}$  with the  $CP$ -violating phase  $\theta$  by assuming  $y_s^{\text{SM}}/x_s^{\text{SM}} = 0.05$  and  $z = 0.5$  (or  $z = 5$ ) in Fig. 1. It is clear that  $\mathcal{A}_{\text{SL}}$  may reach the percent level for favorable values of  $\theta$  and  $z$ . Thus the measurement of  $\mathcal{A}_{\text{SL}}$  at the forthcoming  $B$ -meson factories can provide some useful information or constraints on NP in  $B_s^0$ - $\bar{B}_s^0$  mixing.

Finally it is worth emphasizing that reliable calculations of  $y_s^{\text{SM}}/x_s^{\text{SM}}$  are crucial for probing any NP effect through  $\mathcal{A}_{\text{SL}}$ . A detailed analysis of  $y_s^{\text{SM}}/x_s^{\text{SM}}$  with updated data can be found in [12].

#### 4 NP effect in nonleptonic $B_s$ decays

Let us turn attention to the NP effect on  $CP$  asymmetries in some nonleptonic  $B_s$  decays into  $CP$  eigenstates, such as  $D_s^+ D_s^-$ ,  $D_s^{*+} D_s^- \oplus D_s^+ D_s^{*-}$ ,  $(\psi\phi)_+$  (i.e., the  $CP$ -even state of  $\psi\phi$ ) [4],  $\psi K_S$  and  $\psi K_L$ . These transitions can be used to extract the  $CP$ -violating phase  $\phi_m$ , in most cases, without ambiguities from hadronic matrix elements. They are also favorable in view of the nearest experimental detectability for weak decays of  $B_s$  mesons at  $e^+e^-$  colliders and (or) hadron machines. Obviously,  $q/p = e^{-i2\phi_m}$  should be an excellent approximation.

Due to  $B_s^0$ - $\bar{B}_s^0$  mixing, the time-dependent rates of  $B_s$  mesons decaying into a  $CP$  eigenstate  $f$  can be written as [10]

$$\begin{aligned} & \mathcal{R}[B_{s,\text{phys}}^0(t) \rightarrow f] \\ &= n_f e^{-\Gamma t} \left[ \frac{1 + |\lambda_f|^2}{2} \cosh(y_s \Gamma t) - \text{Re} \lambda_f \sinh(y_s \Gamma t) \right. \\ & \quad \left. + \frac{1 - |\lambda_f|^2}{2} \cos(x_s \Gamma t) - \text{Im} \lambda_f \sin(x_s \Gamma t) \right], \\ & \mathcal{R}[\bar{B}_{s,\text{phys}}^0(t) \rightarrow f] \end{aligned}$$

$$\begin{aligned} &= n_f e^{-\Gamma t} \left[ \frac{1 + |\lambda_f|^2}{2} \cosh(y_s \Gamma t) - \text{Re} \lambda_f \sinh(y_s \Gamma t) \right. \\ & \quad \left. - \frac{1 - |\lambda_f|^2}{2} \cos(x_s \Gamma t) + \text{Im} \lambda_f \sin(x_s \Gamma t) \right], \end{aligned} \quad (16)$$

where  $n_f$  denotes the normalization factor proportional to  $|\langle f | \mathcal{H}_{\text{eff}} | B_s^0 \rangle|^2$ , and  $\lambda_f$  stands for a rephasing-invariant quantity defined by

$$\lambda_f \equiv e^{-i2\phi_m} \frac{\langle f | \mathcal{H}_{\text{eff}} | \bar{B}_s^0 \rangle}{\langle f | \mathcal{H}_{\text{eff}} | B_s^0 \rangle}. \quad (17)$$

We see that the  $x_s$ -induced oscillations, anticipated to be rapid in the SM, cancel in the rate of the flavor-untagged data samples:

$$\begin{aligned} \mathcal{S}(t) &\equiv \mathcal{R}[B_{s,\text{phys}}^0(t) \rightarrow f] + \mathcal{R}[\bar{B}_{s,\text{phys}}^0(t) \rightarrow f], \\ &= n_f e^{-\Gamma t} \left[ (1 + |\lambda_f|^2) \cosh(y_s \Gamma t) \right. \\ & \quad \left. - 2 \text{Re} \lambda_f \sinh(y_s \Gamma t) \right]. \end{aligned} \quad (18)$$

In contrast, the  $y_s$ -induced oscillations disappear in the rate asymmetry between the flavor-tagged data sample:

$$\begin{aligned} \mathcal{A}(t) &\equiv \mathcal{R}[B_{s,\text{phys}}^0(t) \rightarrow f] - \mathcal{R}[\bar{B}_{s,\text{phys}}^0(t) \rightarrow f], \\ &= n_f e^{-\Gamma t} \left[ (1 - |\lambda_f|^2) \cos(x_s \Gamma t) \right. \\ & \quad \left. - 2 \text{Im} \lambda_f \sin(x_s \Gamma t) \right]. \end{aligned} \quad (19)$$

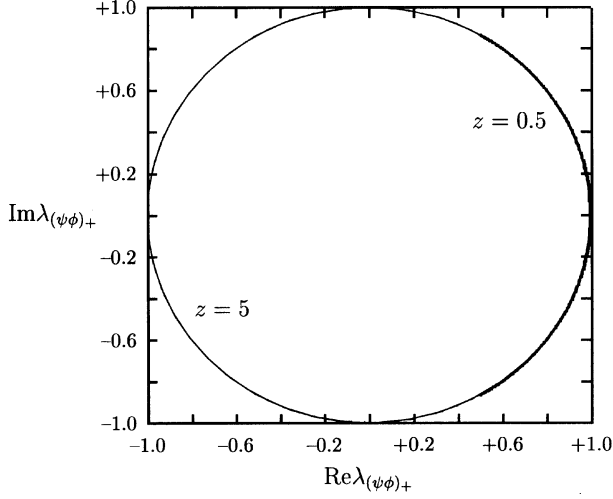
Therefore, measurements of  $\mathcal{S}(t)$  and (or)  $\mathcal{A}(t)$  allow one to determine  $\text{Re} \lambda_f$  versus  $(1 + |\lambda_f|^2)$  and (or)  $\text{Im} \lambda_f$  versus  $(1 - |\lambda_f|^2)$  in most cases, except the special situation where  $y_s$  is vanishingly small (e.g.,  $y_s \leq 10^{-2}$ ) due to NP and  $x_s$  is considerably large (e.g.,  $x_s > 50$ ).

Within the SM,  $\lambda_{D_s^+ D_s^-} \approx 1$ ,  $\lambda_{(\psi\phi)_+} \approx 1$ ,  $\lambda_{\psi K_S} \approx -1$  and  $\lambda_{\psi K_L} \approx 1$  hold as a good approximation. For these decay modes, the strong or electroweak penguin contributions (dominated by the top quark in the loop) are significantly suppressed in magnitude and do not give rise to additional  $CP$ -violating phases [13]. Beyond the SM, the NP appearing in  $B_s^0$ - $\bar{B}_s^0$  mixing is likely to simultaneously manifest itself in the loop-induced penguin amplitudes. Generally we do not expect that the NP-enhanced penguin contributions can be significant enough in those  $B_s$  transitions mentioned above [6], and a test of this assumption is possible by detecting the deviation of  $|\lambda_f|$  from unity in  $\mathcal{A}(t)$ . Here we assume  $|\lambda_f| \approx 1$  to hold even in the presence of NP, for  $B_s \rightarrow D_s^+ D_s^-$ ,  $(\psi\phi)_+$ ,  $\psi K_S$  and  $\psi K_L$ . Then we obtain

$$\begin{aligned} \text{Re} \lambda_{D_s^+ D_s^-} &\approx \text{Re} \lambda_{(\psi\phi)_+} \approx -\text{Re} \lambda_{\psi K_S} \\ &\approx \text{Re} \lambda_{\psi K_L} \approx \cos(2\phi_m), \\ \text{Im} \lambda_{D_s^+ D_s^-} &\approx \text{Im} \lambda_{(\psi\phi)_+} \approx -\text{Im} \lambda_{\psi K_S} \\ &\approx \text{Im} \lambda_{\psi K_L} \approx -\sin(2\phi_m), \end{aligned} \quad (20)$$

where

$$\begin{aligned} \cos(2\phi_m) &= \frac{1 + 2z \cos \theta + z^2 \cos(2\theta)}{1 + z^2 + 2z \cos \theta}, \\ \sin(2\phi_m) &= \frac{2z \sin \theta (1 + z \cos \theta)}{1 + z^2 + 2z \cos \theta}. \end{aligned} \quad (21)$$



**Fig. 2.** Illustrative plot for the correlation between two observables  $\text{Re}\lambda_{(\psi\phi)_+}$  and  $\text{Im}\lambda_{(\psi\phi)_+}$ , where  $\theta \in [-\pi, +\pi]$  and  $z = 0.5$  (dark solid curve) or  $z = 5$  (solid circle) have been taken

Through this way, we can determine the phase information of NP in  $B_s^0\text{-}\bar{B}_s^0$  mixing. To illustrate the correlation between two observables, e.g.,  $\text{Re}\lambda_{(\psi\phi)_+}$  and  $\text{Im}\lambda_{(\psi\phi)_+}$ , we plot them in Fig. 2 by taking  $\theta \in [-\pi, +\pi]$  and  $z = 0.5$  or  $z = 5$ . Clearly the two-fold ambiguity in extracting  $\phi_m$  from  $\text{Re}\lambda_{(\psi\phi)_+} \approx \cos(2\phi_m)$  can be removed if  $\text{Im}\lambda_{(\psi\phi)_+} \approx -\sin(2\phi_m)$  is also measured.

It is obvious that a determination of  $\phi_m$  itself cannot isolate the magnitude ( $z$ ) and phase ( $\theta$ ) of NP effects in  $B_s^0\text{-}\bar{B}_s^0$  mixing. For this reason, combining the measurements of  $\mathcal{A}_{\text{SL}}$ ,  $\mathcal{S}(t)$  and (or)  $\mathcal{A}(t)$  is useful to determine or constrain  $z$  and  $\theta$  separately.

## 5 NP effect in $CP$ -forbidden $B_s^0\bar{B}_s^0$ decays

If the expected large size of  $x_s^{\text{SM}}$  cannot be remarkably reduced by the contribution from NP in  $B_s^0\text{-}\bar{B}_s^0$  mixing, then a study of  $CP$ -forbidden  $B_s^0\bar{B}_s^0$  decays at the  $\Upsilon(5S)$  resonance will become interesting. The straightforward reason is that the rates of such  $CP$ -forbidden transitions are not suppressed by the largeness of  $x_s$ , as we shall show subsequently. The  $CP$ -violating signal may be established directly from the observation of a  $CP$ -forbidden decay mode itself other than the decay rate asymmetry, thus neither flavor tagging nor time-dependent measurements are necessary in practical experiments [9]. For a  $B$ -meson factory running at the  $\Upsilon(5S)$  resonance, one will have a chance to detect the typical  $CP$ -forbidden channels like  $(B_s^0\bar{B}_s^0)_{C=+1} \rightarrow (\psi K_S)(\psi K_L)$  and  $(B_s^0\bar{B}_s^0)_{C=-1} \rightarrow (\psi K_S)(\psi K_S)$  or  $(\psi K_L)(\psi K_L)$ , where  $CP$  parities of the initial and final states are opposite. For simplicity and illustration, we concentrate only on the coherent decays of  $(B_s^0\bar{B}_s^0)_{C=-1}$  events later on.

The time-independent rate of a  $(B_s^0\bar{B}_s^0)_{C=-1}$  pair decaying into the final state  $(f_1 f_2)$  can be given by [9, 10]

$$\mathcal{R}(f_1, f_2) = n_{12} \left\{ \frac{x_s^2}{1 + x_s^2} [1 + |\lambda_{f_1}|^2 |\lambda_{f_2}|^2] \right.$$

$$\begin{aligned} & -2\text{Re}(\lambda_{f_1} \lambda_{f_2}) \\ & + \frac{2 + x_s^2}{1 + x_s^2} [|\lambda_{f_1}|^2 + |\lambda_{f_2}|^2] \\ & \left. - 2\text{Re}(\lambda_{f_1} \lambda_{f_2}^*) \right\}, \end{aligned} \quad (22)$$

where  $n_{12}$  is a normalization factor proportional to the product of  $\mathcal{R}(B_s^0 \rightarrow f_1)$  and  $\mathcal{R}(B_s^0 \rightarrow f_2)$ ; and  $\lambda_{f_i}$  ( $i = 1$  or  $2$ ) is defined like  $\lambda_f$  in (17). In obtaining the above formula, we have made a safe approximation:  $1/(1 - y_s^2) \approx 1$  due to  $y_s \leq y_s^{\text{SM}} \sim 0.1$ . For those  $B_s$  decays into hadronic  $CP$  eigenstates via quark subprocesses  $b \rightarrow (c\bar{c})s$  and  $b \rightarrow (c\bar{c})d$ ,  $|\lambda_{f_1}| \approx |\lambda_{f_2}| \approx 1$  is expected to hold to a good degree of accuracy. With the help of  $\mathcal{R}(B_s^0 \rightarrow \psi K_S) \approx \mathcal{R}(B_s^0 \rightarrow \psi K_L)$ , one can calculate the following ratios of two joint decay rates:

$$\begin{aligned} \xi_{1K} &\equiv \frac{\mathcal{R}(D_s^+ D_s^-, \psi K_L)}{\mathcal{R}(D_s^+ D_s^-, \psi K_S)}, \\ \xi_{2K} &\equiv \frac{\mathcal{R}(\psi K_S, \psi K_S)}{\mathcal{R}(\psi K_S, \psi K_L)}. \end{aligned} \quad (23)$$

The results explicitly read

$$\begin{aligned} \xi_{1K} &= \frac{x_s^2 \left[ 1 - \text{Re}(\lambda_{D_s^+ D_s^-} \lambda_{\psi K_L}) \right] + (2 + x_s^2) \left[ 1 - \text{Re}(\lambda_{D_s^+ D_s^-} \lambda_{\psi K_S}^*) \right]}{x_s^2 \left[ 1 - \text{Re}(\lambda_{D_s^+ D_s^-} \lambda_{\psi K_S}) \right] + (2 + x_s^2) \left[ 1 - \text{Re}(\lambda_{D_s^+ D_s^-} \lambda_{\psi K_S}^*) \right]}, \\ &= \frac{x_s^2 \sin^2(2\phi_m - \phi_K) + (2 + x_s^2) \sin^2 \phi_K}{x_s^2 \cos^2(2\phi_m - \phi_K) + (2 + x_s^2) \cos^2 \phi_K}; \end{aligned} \quad (24)$$

and

$$\begin{aligned} \xi_{2K} &= \frac{x_s^2 [1 - \text{Re}(\lambda_{\psi K_S}^2)] + (2 + x_s^2) [1 - \text{Re}(\lambda_{\psi K_S} \lambda_{\psi K_S}^*)]}{x_s^2 [1 - \text{Re}(\lambda_{\psi K_S} \lambda_{\psi K_L})] + (2 + x_s^2) [1 - \text{Re}(\lambda_{\psi K_S} \lambda_{\psi K_L}^*)]}, \\ &= \frac{x_s^2 \sin^2[2(\phi_m - \phi_K)]}{x_s^2 \cos^2[2(\phi_m - \phi_K)] + (2 + x_s^2)}, \end{aligned} \quad (25)$$

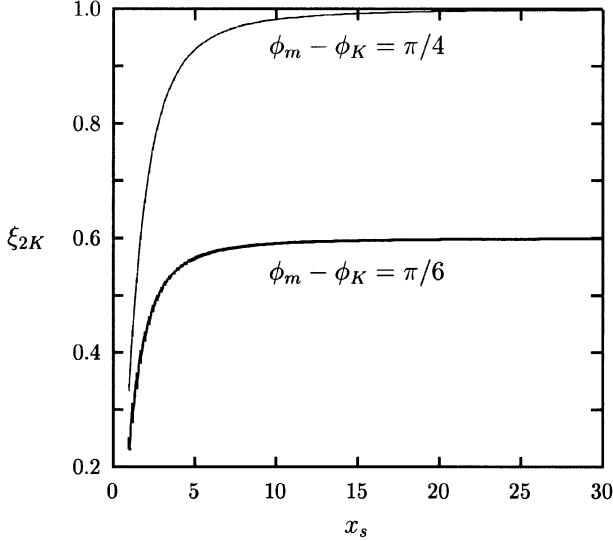
where the phase parameter  $\phi_K$  comes from  $K^0\text{-}\bar{K}^0$  mixing in the final states of  $B_s \rightarrow \psi K_S$  and  $\psi K_L$ :  $2\phi_K \equiv -\arg(q_K/p_K)$ . Within the SM,  $\phi_K \approx 0$  is an excellent approximation. However, the NP appearing in  $B_s^0\text{-}\bar{B}_s^0$  mixing is also likely to manifest itself in  $K^0\text{-}\bar{K}^0$  mixing, leading probably to a small  $CP$ -violating phase  $\phi_K$  (see, e.g., [8]). One can see that the above two  $CP$ -violating signals are not suppressed by the largeness of  $x_s$ . For  $x_s \geq 15$  [1], we approximate (24) and (25) to

$$\begin{aligned} \xi_{1K} &\approx \frac{\sin^2(2\phi_m - \phi_K) + \sin^2 \phi_K}{\cos^2(2\phi_m - \phi_K) + \cos^2 \phi_K}, \\ \xi_{2K} &\approx \frac{\sin^2[2(\phi_m - \phi_K)]}{\cos^2[2(\phi_m - \phi_K)] + 1}. \end{aligned} \quad (26)$$

Clearly the information on  $\phi_m$  and  $\phi_K$  is extractable from  $\xi_{1K}$  and  $\xi_{2K}$ . If  $\phi_K \approx 0$  is further taken [8], then (26) is simplified as

$$\xi_{1K} \approx \xi_{2K} \approx \frac{\sin^2(2\phi_m)}{1 + \cos^2(2\phi_m)}, \quad (27)$$

which is a pure function of  $\phi_m$ .



**Fig. 3.** Illustrative plot for the  $CP$ -violating signal  $\xi_{2K}$  changing with  $x_s \in [1, 30]$ , where  $(\phi_m - \phi_K) = \pi/6$  or  $\pi/4$  has been taken

For the purpose of illustration, we assume  $\phi_m - \phi_K = \pi/6$  or  $\pi/4$  to plot the change of  $\xi_{2K}$  with  $x_s \in [1, 30]$  in Fig. 3. We find that  $\xi_{2K}$  becomes close to its maximal value only if  $x_s \geq 10$ , for a definite input of  $\phi_m - \phi_K$ . Hence such a  $CP$ -violating signal will be of particular experimental interest, provided NP introduces a significant phase into  $B_s^0-\bar{B}_s^0$  (or  $K^0-\bar{K}^0$ ) mixing.

## 6 Concluding remarks

We have studied some phenomenological possibilities to isolate the NP effect in  $B_s^0-\bar{B}_s^0$  mixing. If  $x_s^{\text{SM}}$  (or  $y_s^{\text{SM}}$ ) can be reliably evaluated and  $x_s$  (or  $y_s$ ) can be experimentally determined, then significant deviation of  $x_s/x_s^{\text{SM}}$  (or  $y_s/y_s^{\text{SM}}$ ) from unity will be a clear signal of NP. In addition, NP in  $B_s^0-\bar{B}_s^0$  mixing is likely to enhance the semileptonic  $CP$  asymmetry  $\mathcal{A}_{\text{SL}}$  to the level of  $10^{-3} - 10^{-2}$ . The phase information of NP can be extracted from either flavor-tagged or flavor-untagged data samples of  $B_s$  decays into hadronic  $CP$  eigenstates, or both of them, depending upon the experimental sensitivity to the  $x_s$ - and (or)  $y_s$ -induced oscillations. It is remarkable that the rates of  $CP$ -forbidden  $B_s^0\bar{B}_s^0$  transitions on the  $\Upsilon(5S)$  resonance are not suppressed by the largeness of  $x_s$ , and this feature will be useful for measuring the  $CP$ -violating phase in  $B_s^0-\bar{B}_s^0$  (or  $K^0-\bar{K}^0$ ) mixing.

Of course, the experimental feasibility of the above-proposed measurements has to be studied in some detail [12]. In particular, the complication to observe coherent  $B_d^0\bar{B}_d^0$  decays at the  $\Upsilon(5S)$  resonance, where coherent  $B_d^0\bar{B}_d^0$  events can also be produced [12,14], should be taken seriously. We hope that a systematic search for various  $CP$ -violating signals in the  $B_s^0-\bar{B}_s^0$  system will be available in the future experiments of  $B$ -meson physics.

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## References

1. L. Gibbons (CLEO Collaboration), invited talk given at the Int. Conf. on HEP, Warsaw, Poland (July 1996)
2. I. Dunietz, Phys. Rev. **D 52** (1995) 3048
3. M. Beneke, G. Buchalla, and I. Dunietz, Phys. Rev. **D 54** (1996) 4419
4. A.S. Dighe, I. Dunietz, H.J. Lipkin, and J.L. Rosner, Phys. Lett. **B 369** (1996) 144
5. Y. Grossman, Phys. Lett. **B 380** (1996) 99
6. I.I. Bigi, V.A. Khoze, N.G. Uraltsev, and A.I. Sanda, in *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1988), p. 175;  
J.L. Hewett, T. Takeuchi, and S. Thomas, SLAC-PUB-7088 or CERN-TH/96-56;  
Y. Grossman, Y. Nir, and R. Rattazzi, SLAC-PUB-7379 or CERN-TH-96-368;  
M. Gronau and D. London, Phys. Rev. **D 55** (1997) 2845
7. Y. Nir, Phys. Lett. **B 327** (1994) 85
8. A.G. Cohen, D.B. Kaplan, F. Lepeintre, and A.E. Nelson, Phys. Rev. Lett. **78** (1997) 2300;  
J.P. Silva and L. Wolfenstein, Phys. Rev. **D 55** (1997) 5331
9. Z.Z. Xing, Phys. Rev. **D 53** (1996) 204
10. Z.Z. Xing, Phys. Rev. **D 55** (1997) 196
11. Z.Z. Xing, Phys. Lett. **B 379** (1996) 257
12. A comprehensive study of this problem is underway
13. Y. Nir and H.R. Quinn, Annu. Rev. Nucl. Part. Sci. **42** (1992) 211
14. P. Krawczyk, D. London, and H. Steger, Nucl. Phys. **B 321** (1989) 1